Question 20, Final, F07

20 Let $A =$

$$\begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}$$

Which of the following gives the entry in the 2nd row and 1st column of A^{-1}?

(a) -1 (b) 3 (c) 1 (d) -2 (e) $\frac{1}{3}$

Question 20, Final, F07

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$$\begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}$$

Which of the following gives the entry in the 2nd row and 1st column of A^{-1}?

(a) -1 (b) 3 (c) 1 (d) -2 (e) $\frac{1}{3}$

- *If ad - bc is not zero, the inverse of the matrix* $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ *is given by*

$$\begin{pmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{pmatrix}$$

Question 20, Final, F07

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$$\begin{pmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{pmatrix}$$

- *Since* $2 \cdot 1 - 1 \cdot 3 = -1 \neq 0$, *the inverse of the matrix* $\begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}$ *is given by*

$$\begin{pmatrix} \frac{1}{-1} & \frac{-3}{-1} \\ \frac{-1}{-1} & \frac{2}{-1} \end{pmatrix} = \begin{pmatrix} -1 & 3 \\ 1 & -2 \end{pmatrix}$$

Question 20, Final, F07

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$$\begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}$$

Which of the following gives the entry in the 2nd row and 1st column of A^{-1}?

(a) -1 (b) 3 (c) 1 (d) -2 (e) $\frac{1}{3}$

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- *Since* $2 \cdot 1 - 1 \cdot 3 = -1 \neq 0$, *the inverse of the matrix* $\begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}$ *is given by*

$$\begin{pmatrix} \frac{1}{-1} & \frac{-3}{-1} \\ \frac{-1}{-1} & \frac{2}{-1} \end{pmatrix} = \begin{pmatrix} -1 & 3 \\ 1 & -2 \end{pmatrix}$$

- *The correct answer is (c).*

Question 21, Final, F07

21 Let

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \\ 0 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 1 \\ 5 & 0 \\ 0 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

Calculate $(A - B) \cdot C$.

(a) $\begin{pmatrix} 0 & 1 \\ -1 & 1 \\ 1 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} -1 & 1 \\ -2 & 1 \\ 0 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 0 \\ 1 & -1 \\ 1 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & -1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ (e) $\begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}$

Question 21, Final, F07

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$$A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \\ 0 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 1 \\ 5 & 0 \\ 0 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

Calculate $(A - B) \cdot C$.

(a) $\begin{pmatrix} 0 & 1 \\ -1 & 1 \\ 1 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} -1 & 1 \\ -2 & 1 \\ 0 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 0 \\ 1 & -1 \\ 1 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & -1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ (e) $\begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}$

- $$A - B = \begin{pmatrix} 1-2 & 2-1 \\ 3-5 & 1-0 \\ 0-0 & 2-1 \end{pmatrix} = \begin{pmatrix} -1 & 1 \\ -2 & 1 \\ 0 & 1 \end{pmatrix}.$$

Question 21, Final, F07

21 Let

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \\ 0 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 1 \\ 5 & 0 \\ 0 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

Calculate $(A - B) \cdot C$.

(a) $\begin{pmatrix} 0 & 1 \\ -1 & 1 \\ 1 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} -1 & 1 \\ -2 & 1 \\ 0 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 0 \\ 1 & -1 \\ 1 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & -1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ (e) $\begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}$

- $A \text{ - } B = \begin{pmatrix} 1-2 & 2-1 \\ 3-5 & 1-0 \\ 0-0 & 2-1 \end{pmatrix} = \begin{pmatrix} -1 & 1 \\ -2 & 1 \\ 0 & 1 \end{pmatrix}.$
- $(A - B) \cdot C = \begin{pmatrix} -1 & 1 \\ -2 & 1 \\ 0 & 1 \end{pmatrix}_{3\times 2} \cdot \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}_{2\times 2} = \begin{pmatrix} -1+1 & 0+1 \\ -2+1 & 0+1 \\ 0+1 & 0+1 \end{pmatrix}_{3\times 2}$

$$= \begin{pmatrix} 0 & 1 \\ -1 & 1 \\ 1 & 1 \end{pmatrix}_{3\times 2}$$

Question 21, Final, F07

21 Let

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \\ 0 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 1 \\ 5 & 0 \\ 0 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

Calculate $(A - B) \cdot C$.

(a) $\begin{pmatrix} 0 & 1 \\ -1 & 1 \\ 1 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} -1 & 1 \\ -2 & 1 \\ 0 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 0 \\ 1 & -1 \\ 1 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & -1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ (e) $\begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}$

- A - $B = \begin{pmatrix} 1-2 & 2-1 \\ 3-5 & 1-0 \\ 0-0 & 2-1 \end{pmatrix} = \begin{pmatrix} -1 & 1 \\ -2 & 1 \\ 0 & 1 \end{pmatrix}.$
- $(A-B) \cdot C = \begin{pmatrix} -1 & 1 \\ -2 & 1 \\ 0 & 1 \end{pmatrix}_{3\times 2} \cdot \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}_{2\times 2} = \begin{pmatrix} -1+1 & 0+1 \\ -2+1 & 0+1 \\ 0+1 & 0+1 \end{pmatrix}_{3\times 2}$
  $= \begin{pmatrix} 0 & 1 \\ -1 & 1 \\ 1 & 1 \end{pmatrix}_{3\times 2}$
- *The correct answer is (a).*

Question 22, Final, F07

22 Let

$$C = \begin{pmatrix} 2 & 1 & 3 \\ 0 & 2 & 4 \end{pmatrix}, \quad D = \begin{pmatrix} 5 & 2 \\ 1 & 0 \\ 2 & 1 \end{pmatrix}.$$

Find the entry in the second row and first column of the matrix $C \cdot D$.

(a) 10 (b) 4 (c) 7 (d) 17 (e) 0

Question 22, Final, F07

22 Let

$$C = \begin{pmatrix} 2 & 1 & 3 \\ 0 & 2 & 4 \end{pmatrix}, \quad D = \begin{pmatrix} 5 & 2 \\ 1 & 0 \\ 2 & 1 \end{pmatrix}.$$

Find the entry in the second row and first column of the matrix $C \cdot D$.

(a) 10 (b) 4 (c) 7 (d) 17 (e) 0

- $C \cdot D = \begin{pmatrix} 2 & 1 & 3 \\ 0 & 2 & 4 \end{pmatrix}_{2\times 3} \cdot \begin{pmatrix} 5 & 2 \\ 1 & 0 \\ 2 & 1 \end{pmatrix}_{3\times 2}$

$$= \begin{pmatrix} - & - \\ 0 \cdot 5 + 2 \cdot 1 + 4 \cdot 2 & - \end{pmatrix}_{2\times 2} = \begin{pmatrix} - & - \\ 10 & - \end{pmatrix}$$

Question 22, Final, F07

22 Let

$$C = \begin{pmatrix} 2 & 1 & 3 \\ 0 & 2 & 4 \end{pmatrix}, \quad D = \begin{pmatrix} 5 & 2 \\ 1 & 0 \\ 2 & 1 \end{pmatrix}.$$

Find the entry in the second row and first column of the matrix $C \cdot D$.

(a) 10 (b) 4 (c) 7 (d) 17 (e) 0

- $C \cdot D = \begin{pmatrix} 2 & 1 & 3 \\ 0 & 2 & 4 \end{pmatrix}_{2\times 3} \cdot \begin{pmatrix} 5 & 2 \\ 1 & 0 \\ 2 & 1 \end{pmatrix}_{3\times 2}$

  $= \begin{pmatrix} - & - \\ 0 \cdot 5 + 2 \cdot 1 + 4 \cdot 2 & - \end{pmatrix}_{2\times 2} = \begin{pmatrix} - & - \\ 10 & - \end{pmatrix}$
- *The correct answer is (a).*

Question 23, Final, F07

23 Let

$$A = \begin{pmatrix} 5 & 2 \\ 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 1 \\ 0 & 2 \\ 1 & -1 \end{pmatrix}, \quad C = \begin{pmatrix} 5 & 2 \\ 1 & 0 \\ 2 & 1 \end{pmatrix}, \quad D = \begin{pmatrix} 2 & 1 & 5 \end{pmatrix}.$$

Which of the following statements is true?

(a) A^{-1} does not exist. *False, A^{-1} does exist because $\det A = 5 - 2 = 3 \neq 0$.*

(b) $C \cdot B$ does not exist. *True, because $C_{3\times 2}$ and $B_{3\times 2}$ do not have compatible dimensions for multiplication.*

(c) $D \cdot C$ does not exist. *False, because $D_{1\times 3}$ and $C_{3\times 2}$ have compatible dimensions to calculate $D \cdot C$.*

(d) $B \cdot A$ does not exist. *False, because $B_{3\times 2}$ and $A_{2\times 2}$ have compatible dimensions to calculate $B \cdot A$.*

(e) $(B - C) \cdot A$ does not exist. *False, because $(B - C)_{3\times 2}$ and $A_{2\times 2}$ have compatible dimensions to calculate $(B - C) \cdot A$.*

Question 24, Final, F07

24 The following matrix is the payoff matrix for the row player in a zero-sum game:

$$\begin{pmatrix} 0 & 1 & 2 \\ -1 & 2 & -2 \\ -1 & 0 & 1 \end{pmatrix}$$

The payoff matrix has a saddle point; where is it?

(*a*) Row 1, Col 1 (*b*) Row 1, Col 3 (*c*) Row 2, Col 3 (*d*) Row 3, Col 1 (*e*) Row 2, Col 2

Question 24, Final, F07

24 The following matrix is the payoff matrix for the row player in a zero-sum game:

$$\begin{pmatrix} 0 & 1 & 2 \\ -1 & 2 & -2 \\ -1 & 0 & 1 \end{pmatrix}$$

The payoff matrix has a saddle point; where is it?

(*a*) Row 1, Col 1 (*b*) Row 1, Col 3 (*c*) Row 2, Col 3 (*d*) Row 3, Col 1 (*e*) Row 2, Col 2

- *We look at the minimum in each row and the maximum of each column and compare:*

				Min.
	0	1	2	0
	−1	2	−2	−2
	−1	0	1	−1
Max.	0	2	2	

Question 24, Final, F07

24 The following matrix is the payoff matrix for the row player in a zero-sum game:

$$\begin{pmatrix} 0 & 1 & 2 \\ -1 & 2 & -2 \\ -1 & 0 & 1 \end{pmatrix}$$

The payoff matrix has a saddle point; where is it?

(*a*) Row 1, Col 1 (*b*) Row 1, Col 3 (*c*) Row 2, Col 3 (*d*) Row 3, Col 1 (*e*) Row 2, Col 2

- *We look at the minimum in each row and the maximum of each column and compare:*

				Min.
	0	1	2	0
	−1	2	−2	−2
	−1	0	1	−1
Max.	0	2	2	

- *The entry in row 1 and column 1 is the minimum in its row and the maximum in its column, hence it is a saddle point.*

Question 24, Final, F07

24 The following matrix is the payoff matrix for the row player in a zero-sum game:

$$\begin{pmatrix} 0 & 1 & 2 \\ -1 & 2 & -2 \\ -1 & 0 & 1 \end{pmatrix}$$

The payoff matrix has a saddle point; where is it?

(*a*) Row 1, Col 1 (*b*) Row 1, Col 3 (*c*) Row 2, Col 3 (*d*) Row 3, Col 1 (*e*) Row 2, Col 2

- *We look at the minimum in each row and the maximum of each column and compare:*

				Min.
	0	1	2	0
	−1	2	−2	−2
	−1	0	1	−1
Max.	0	2	2	

- *The entry in row 1 and column 1 is the minimum in its row and the maximum in its column, hence it is a saddle point.*
- *The correct answer is (a).*

Question 25, Final, F07

25 Roadrunner (R) and Coyote (C) play a game. They each have 4 cards, numbered 1, 2, 3 and 4. They each display one card simultaneously. If both numbers are even Coyote gives Roadrunner $1.If both numbers are odd, Roadrunner gives Coyote $1. If the numbers are neither both even nor both odd, the creature displaying the higher number receives $1 from the other creature. Which of the following payoff matrices gives the payoff matrix for Roadrunner for this game?

(a)

Card #	C 1	2	3	4
1	−1	1	1	1
2	−1	1	1	1
R 3	−1	−1	1	1
4	−1	−1	−1	−1

(b)

Card #	C 1	2	3	4
1	1	1	−1	−1
2	1	1	−1	−1
R 3	1	1	1	−1
4	1	1	1	1

(c)

Card #	C 1	2	3	4
1	1	0	0	1
2	0	1	2	3
R 3	0	0	1	2
4	1	1	−1	−1

(d)

Card #	C 1	2	3	4
1	−1	0	1	1
2	0	−1	1	−1
R 3	0	0	−1	−1
4	1	1	−1	−1

(e)

Card #	C 1	2	3	4
1	−1	−1	−1	−1
2	1	1	−1	1
R 3	−1	1	−1	−1
4	1	1	1	1

Question 25, Final, F07

25 Roadrunner (R) and Coyote (C) play a game. They each have 4 cards, numbered 1, 2, 3 and 4. They each display one card simultaneously. If both numbers are even Coyote gives Roadrunner $1.If both numbers are odd, Roadrunner gives Coyote $1. If the numbers are neither both even nor both odd, the creature displaying the higher number receives $1 from the other creature. Which of the following payoff matrices gives the payoff matrix for Roadrunner for this game?

(a)

	Card #	C 1	2	3	4
	1	−1	1	1	1
	2	−1	1	1	1
R	3	−1	−1	1	1
	4	−1	−1	−1	−1

(b)

	Card #	C 1	2	3	4
	1	1	1	−1	−1
	2	1	1	−1	−1
R	3	1	1	1	−1
	4	1	1	1	1

(c)

	Card #	C 1	2	3	4
	1	1	0	0	1
	2	0	1	2	3
R	3	0	0	1	2
	4	1	1	−1	−1

(d)

	Card #	C 1	2	3	4
	1	−1	0	1	1
	2	0	−1	1	−1
R	3	0	0	−1	−1
	4	1	1	−1	−1

(e)

	Card #	C 1	2	3	4
	1	−1	−1	−1	−1
	2	1	1	−1	1
R	3	−1	1	−1	−1
	4	1	1	1	1

- *The correct answer is (e). by comparing with the instructions.*

Question 26, Final, F07

26 Rat (R) and Cat (C) play a zero-sum game with payoff matrix for Rat given below. What is the optimal pure strategy for Cat for this game?

$$\begin{pmatrix} 1 & 0 & 0 & 2 & 1 \\ 2 & 1 & 0 & 1 & 2 \\ 3 & 2 & -1 & 4 & 6 \\ -1 & -2 & 1 & -1 & -2 \\ 0 & 1 & -1 & 0 & -5 \end{pmatrix}$$

(a) Col 1 (b) Col 2 (c) Col 3 (d) Col 4 (e) Col 5

Question 26, Final, F07

26 Rat (R) and Cat (C) play a zero-sum game with payoff matrix for Rat given below. What is the optimal pure strategy for Cat for this game?

	1	0	0	2	1
	2	1	0	1	2
	3	2	−1	4	6
	−1	−2	1	−1	−2
	0	1	−1	0	−5
Max	3	2	1	4	6

(*a*) Col 1 (*b*) Col 2 (*c*) Col 3 (*d*) Col 4 (*e*) Col 5

- *We calculate the max. of each column and then choose the minimum of these to give Col 3.*

Question 26, Final, F07

26 Rat (R) and Cat (C) play a zero-sum game with payoff matrix for Rat given below. What is the optimal pure strategy for Cat for this game?

	1	0	0	2	1
	2	1	0	1	2
	3	2	−1	4	6
	−1	−2	1	−1	−2
	0	1	−1	0	−5
Max	3	2	1	4	6

(*a*) Col 1 (*b*) Col 2 (*c*) Col 3 (*d*) Col 4 (*e*) Col 5

- *We calculate the max. of each column and then choose the minimum of these to give Col 3.*
- *The correct answer is (c).*

Question 27, Final, F07

27 Catman (C) and Robin (R) play a zero-sum game, with payoff matrix for Robin given by

$$\begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$$

If Robin plays the mixed strategy (.8 .2) and Catman plays the mixed strategy $\begin{pmatrix} .6 \\ .4 \end{pmatrix}$ What is the expected payoff for Robin for the game?

(a) 1.4 (b) 1.48 (c) 1.6 (d) .5 (e) .8

Question 27, Final, F07

27 Catman (C) and Robin (R) play a zero-sum game, with payoff matrix for Robin given by

$$\begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$$

If Robin plays the mixed strategy (.8 .2) and Catman plays the mixed strategy $\begin{pmatrix} .6 \\ .4 \end{pmatrix}$ What is the expected payoff for Robin for the game?

(a) 1.4 (b) 1.48 (c) 1.6 (d) .5 (e) .8

- *The expected pay-off for Robin is given by the product:*

$$(0.8 \quad 0.2)\begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}\begin{pmatrix} .6 \\ .4 \end{pmatrix} = (0.8 + 0.6 \quad 1.6 + 0)\begin{pmatrix} .6 \\ .4 \end{pmatrix}$$

$$= (1.4 \quad 1.6)\begin{pmatrix} .6 \\ .4 \end{pmatrix} = ((1.4)(0.6) + (1.6)(0.4)) = 1.48$$

Question 27, Final, F07

27 Catman (C) and Robin (R) play a zero-sum game, with payoff matrix for Robin given by

$$\begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$$

If Robin plays the mixed strategy (.8 .2) and Catman plays the mixed strategy $\begin{pmatrix} .6 \\ .4 \end{pmatrix}$ What is the expected payoff for Robin for the game?

(*a*) 1.4 (*b*) 1.48 (*c*) 1.6 (*d*) .5 (*e*) .8

- *The expected pay-off for Robin is given by the product:*

$$\begin{pmatrix} 0.8 & 0.2 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix} \begin{pmatrix} .6 \\ .4 \end{pmatrix} = \begin{pmatrix} 0.8 + 0.6 & 1.6 + 0 \end{pmatrix} \begin{pmatrix} .6 \\ .4 \end{pmatrix}$$

$$= \begin{pmatrix} 1.4 & 1.6 \end{pmatrix} \begin{pmatrix} .6 \\ .4 \end{pmatrix} = ((1.4)(0.6) + (1.6)(0.4)) = 1.48$$

- *The correct answer is (b).*

28 Suppose the payoff matrix for Robin, in a zero sum game with Catman, is as in the previous problem:

$$\begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$$

If Robin plays the mixed strategy (.8 .2), which of the following mixed strategies should Catman play to maximize his (Catman's) expected payoff in the game?

(a) $\begin{pmatrix} .6 \\ .4 \end{pmatrix}$ (b) $\begin{pmatrix} .4 \\ .6 \end{pmatrix}$ (c) $\begin{pmatrix} .3 \\ .7 \end{pmatrix}$ (d) $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ (e) $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

28 Suppose the payoff matrix for Robin, in a zero sum game with Catman, is as in the previous problem:

$$\begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$$

If Robin plays the mixed strategy (.8 .2), which of the following mixed strategies should Catman play to maximize his (Catman's) expected payoff in the game?

(a) $\begin{pmatrix} .6 \\ .4 \end{pmatrix}$ (b) $\begin{pmatrix} .4 \\ .6 \end{pmatrix}$ (c) $\begin{pmatrix} .3 \\ .7 \end{pmatrix}$ (d) $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ (e) $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

- *For each of the strategies for Catman listed above,* $\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$*, if Robin plays* (.8 .2)*, the expected pay-off for Robin will be*

$$\begin{pmatrix} 0.8 & 0.2 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 1.4 & 1.6 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = (1.4c_1 + 1.6c_2)$$

28 Suppose the payoff matrix for Robin, in a zero sum game with Catman, is as in the previous problem:

$$\begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$$

If Robin plays the mixed strategy (.8 .2), which of the following mixed strategies should Catman play to maximize his (Catman's) expected payoff in the game?

(a) $\begin{pmatrix} .6 \\ .4 \end{pmatrix}$ (b) $\begin{pmatrix} .4 \\ .6 \end{pmatrix}$ (c) $\begin{pmatrix} .3 \\ .7 \end{pmatrix}$ (d) $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ (e) $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

- *For each of the strategies for Catman listed above,* $\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$, *if Robin plays* (.8 .2), *the expected pay-off for Robin will be*

$$(0.8 \quad 0.2) \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = (1.4 \quad 1.6) \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = (1.4c_1 + 1.6c_2)$$

- *Comparing the values for the strategies given above for Catman, we find the minimum expected pay-off for Robin, which gives the maximum expected pay-off for Catman.*

$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$	$1.4c_1 + 1.6c_2$
$\begin{pmatrix} 0.6 \\ 0.4 \end{pmatrix}$	$1.4(0.6) + 1.6(0.4) = 1.48$
$\begin{pmatrix} 0.4 \\ 0.6 \end{pmatrix}$	$1.4(0.4) + 1.6(0.6) = 1.52$
$\begin{pmatrix} 0.3 \\ 0.7 \end{pmatrix}$	$1.4(0.3) + 1.6(0.7) = 1.54$

$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$	$1.4c_1 + 1.6c_2$
$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$1.4(1) + 1.6(0) = 1.4$(*min : correct answer is* (*d*)))
$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	$1.4(0) + 1.6(1) = 1.6$

29 Rapunzel (R) and Cinderella (C) play a zero-sum game with payoff matrix for Rapunzel given by

$$\begin{pmatrix} 5 & 2 \\ 1 & 3 \end{pmatrix}.$$

Rapunzel wants to find the optimal mixed strategy, assuming that Cinderella always plays the best counterstrategy. Which of the following linear programming problems must she solve:

(a) minimize $x + y$
constraints $x \geq 0, \quad y \geq 0$
$x + 5y \geq 1$
$3x + 2y \geq 1$

(b) maximize $x + y$
constraints $x \geq 0, \quad y \geq 0$
$5x + y \geq 1$
$2x + 3y \geq 1$

(c) minimize $x + y$
constraints $x \geq 0, \quad y \geq 0$
$5x + y \leq 1$
$2x + 3y \leq 1$

(d) maximize $x + y$
constraints $x \geq 0, \quad y \geq 0$
$x + 5y \leq 1$
$3x + 2y \leq 1$

(e) minimize $x + y$
constraints $x \geq 0, \quad y \geq 0$
$5x + y \geq 1$
$2x + 3y \geq 1$

29 Rapunzel (R) and Cinderella (C) play a zero-sum game with payoff matrix for Rapunzel given by

$$\begin{pmatrix} 5 & 2 \\ 1 & 3 \end{pmatrix}.$$

Rapunzel wants to find the optimal mixed strategy, assuming that Cinderella always plays the best counterstrategy. Which of the following linear programming problems must she solve:

(a) minimize $x + y$; constraints $x \geq 0$, $y \geq 0$, $x + 5y \geq 1$, $3x + 2y \geq 1$

(b) maximize $x + y$; constraints $x \geq 0$, $y \geq 0$, $5x + y \geq 1$, $2x + 3y \geq 1$

(c) minimize $x + y$; constraints $x \geq 0$, $y \geq 0$, $5x + y \leq 1$, $2x + 3y \leq 1$

(d) maximize $x + y$; constraints $x \geq 0$, $y \geq 0$, $x + 5y \leq 1$, $3x + 2y \leq 1$

(e) minimize $x + y$; constraints $x \geq 0$, $y \geq 0$, $5x + y \geq 1$, $2x + 3y \geq 1$

- *The linear programming problem associated with finding Rapunzel's best mixed strategy is summarized in the form :*
  Minimize $x + y$ *subject to the constraints:* $x \geq 0$, $y \geq 0$ *and*

$$\begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 5 & 2 \\ 1 & 3 \end{pmatrix} \geq \begin{pmatrix} 1 & 1 \end{pmatrix}.$$

29 Rapunzel (R) and Cinderella (C) play a zero-sum game with payoff matrix for Rapunzel given by

$$\begin{pmatrix} 5 & 2 \\ 1 & 3 \end{pmatrix}.$$

Rapunzel wants to find the optimal mixed strategy, assuming that Cinderella always plays the best counterstrategy. Which of the following linear programming problems must she solve:

(a) minimize $x + y$; constraints $x \geq 0$, $y \geq 0$, $x + 5y \geq 1$, $3x + 2y \geq 1$

(b) maximize $x + y$; constraints $x \geq 0$, $y \geq 0$, $5x + y \geq 1$, $2x + 3y \geq 1$

(c) minimize $x + y$; constraints $x \geq 0$, $y \geq 0$, $5x + y \leq 1$, $2x + 3y \leq 1$

(d) maximize $x + y$; constraints $x \geq 0$, $y \geq 0$, $x + 5y \leq 1$, $3x + 2y \leq 1$

(e) minimize $x + y$; constraints $x \geq 0$, $y \geq 0$, $5x + y \geq 1$, $2x + 3y \geq 1$

- *The linear programming problem associated with finding Rapunzel's best mixed strategy is summarized in the form :*
  Minimize $x + y$ *subject to the constraints:* $x \geq 0, \quad y \geq 0$ *and*

$$\begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 5 & 2 \\ 1 & 3 \end{pmatrix} \geq \begin{pmatrix} 1 & 1 \end{pmatrix}.$$

- *The resulting optimization problem is :*

(e) *minimize* $x + y$; *constraints* $x \geq 0$, $y \geq 0$, $5x + y \geq 1$, $2x + 3y \geq 1$

Question 30, Final, F07

30 If Rapunzel found that the solution to the linear programming problem for Question 29 was

$$x = \frac{2}{13}, \quad y = \frac{3}{13},$$

what would her optimal mixed strategy be?

(a) $\left(\frac{4}{5}, \frac{1}{5}\right)$ (b) $\left(\frac{2}{5}, \frac{3}{5}\right)$ (c) $\left(\frac{3}{5}, \frac{2}{5}\right)$ (d) $\left(\frac{10}{13}, \frac{3}{13}\right)$ (e) $(0, 1)$

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- *The optimal mixed strategy for Rapunzel is given by*

$$\begin{pmatrix} r_1 & r_2 \end{pmatrix} = \begin{pmatrix} \frac{x}{x+y} & \frac{y}{x+y} \end{pmatrix} = \begin{pmatrix} \frac{\frac{2}{13}}{\frac{2}{13}+\frac{3}{13}} & \frac{\frac{3}{13}}{\frac{2}{13}+\frac{3}{13}} \end{pmatrix} = \begin{pmatrix} \frac{2}{5} & \frac{3}{5} \end{pmatrix}$$

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- *The correct answer is (b).*